

Towards direct detection of Exo-planets

using the Electric Field Conjugation Algorithm

Amir Give'on
Jet Propulsion Laboratory

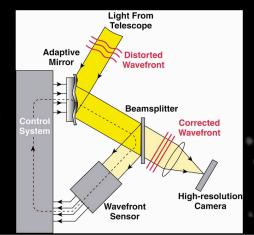
Collaborators:

JPL: Brian Kern, Stuart Shaklan, Dwight Moody.

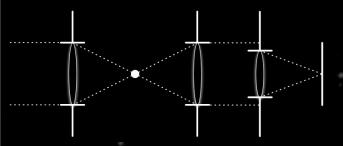
Game plan

- I am going to show that all correction algorithms can be written as a solution to the same equation (the devil is in the details).
- I will explain what EFC is.
- I will show the best DM diversity-based reconstruction algorithm we have so far.
- I will show experimental results for two types of coronagraphs to show the flexibility of the algorithm.









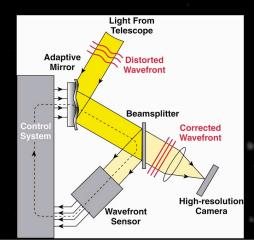
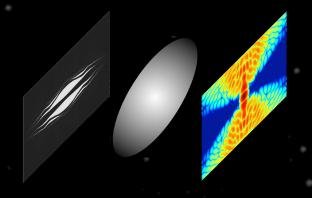
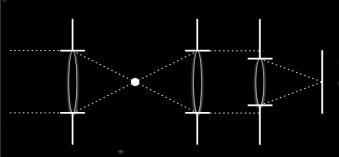
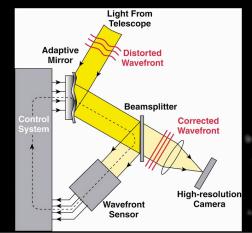


Figure by Claire Max

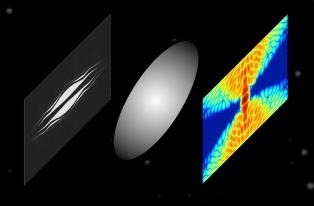


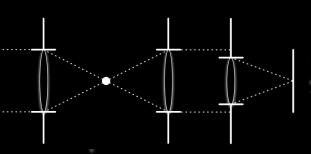


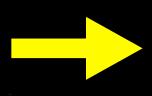


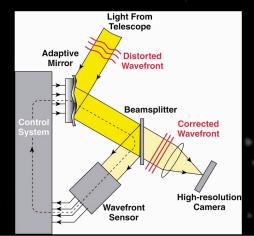




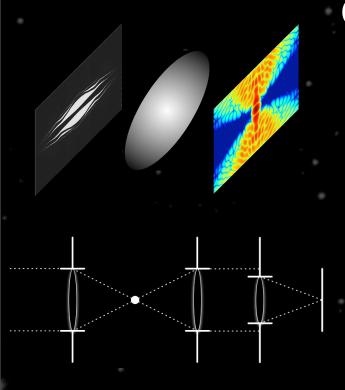


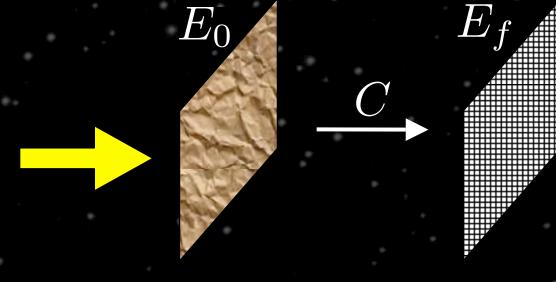


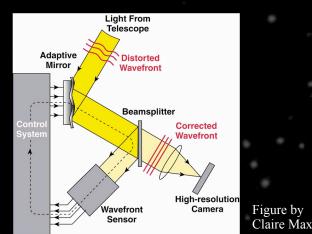










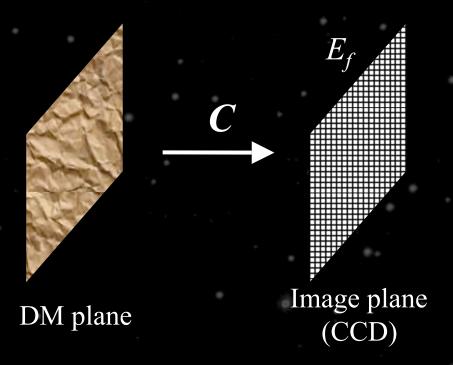


DM plane

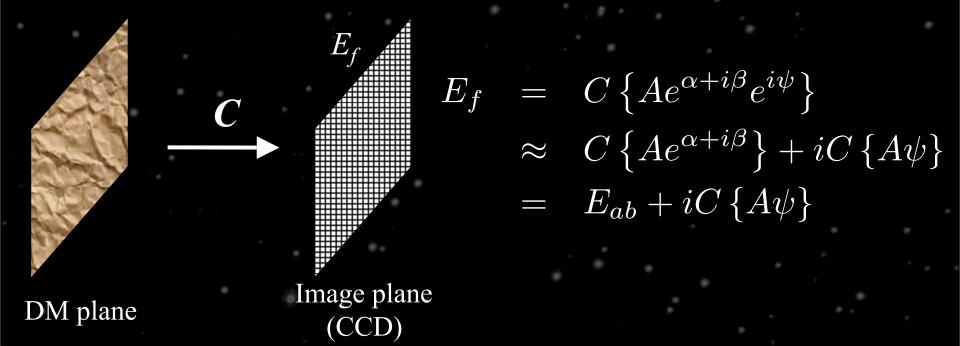
Affected plane

$$E_f = C\left\{E_0\right\}$$

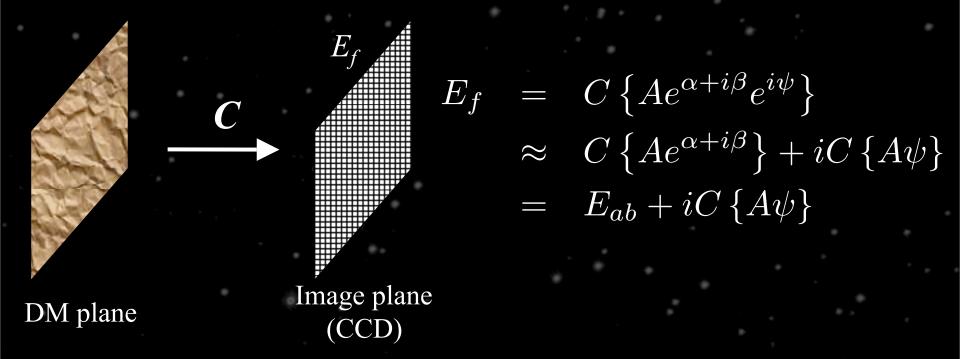








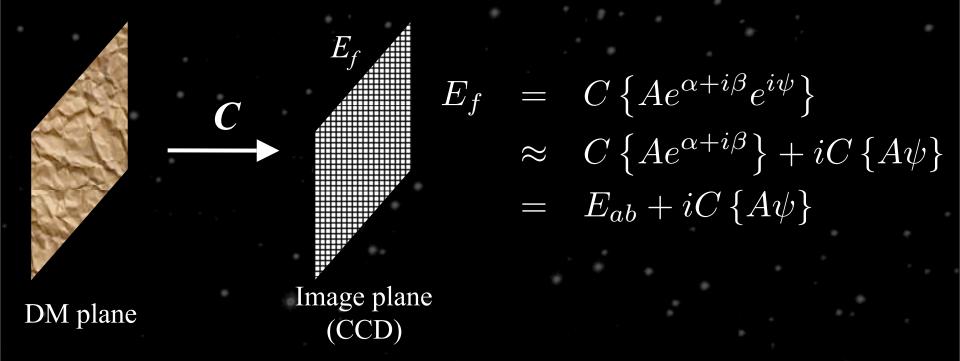




Assuming the effect of the DM can be modeled as the sum of the effects of the individual influence functions:

$$C\left\{A\psi\right\} = G\bar{a}$$





Assuming the effect of the DM can be modeled as the sum of the effects of the individual influence functions:

$$C\left\{A\psi\right\} = G\bar{a}$$

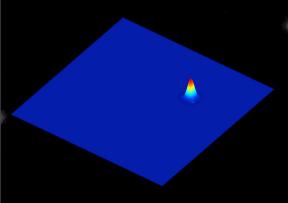
Then, in order to null the electric field in the image plane with the DM:

$$G\bar{a} = iE_{ab}$$

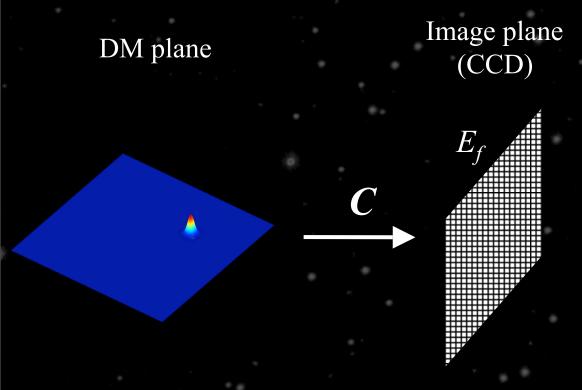


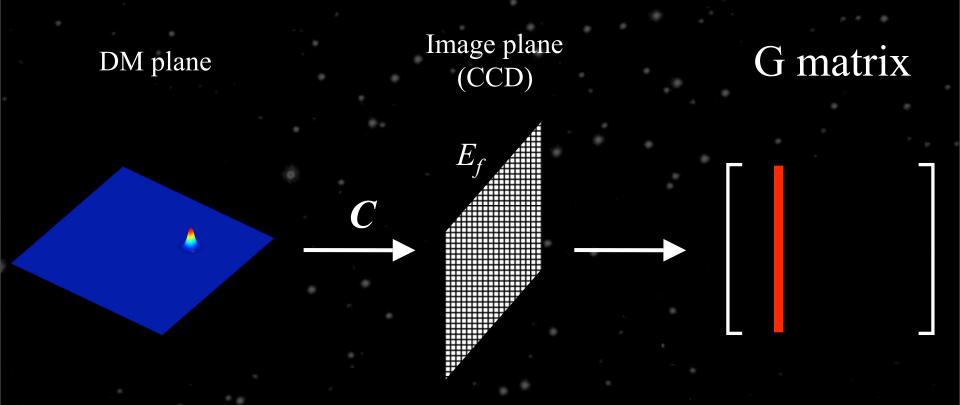
What is this G matrix?

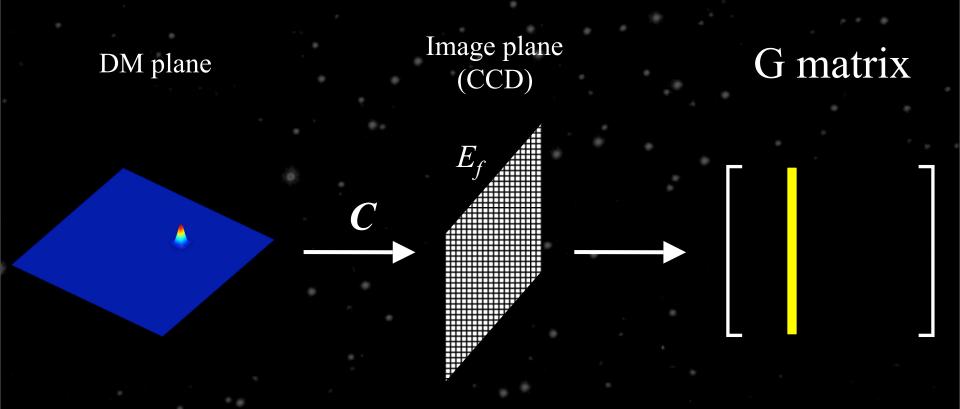
DM plane

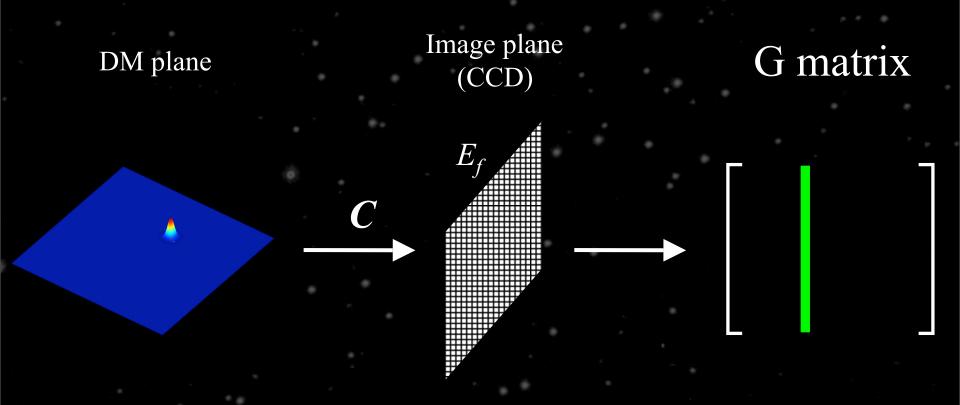






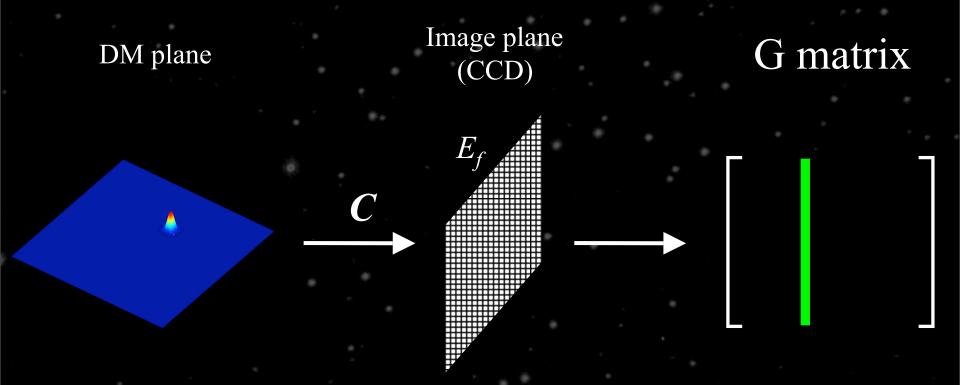








What is this *G* matrix?



The main assumption in this method is that the overall effect of the DM in the image plane is the sum of the effects of each of the actuators



Solving $G\bar{a} = iE_{ab}$





complex valued



Solving
$$G\bar{a}=iE_{ab}$$
 complex valued

Problem: \bar{a} must be real valued.



Solving
$$G\bar{a} = iE_{ab}$$
complex valued

Problem: \bar{a} must be real valued.

Solution:

$$ar{a} = \left[egin{array}{c} \Re \left\{ G
ight\} \ \cdots \ \Im \left\{ G
ight\} \end{array} \right]^{-1} \left[egin{array}{c} \Re \left\{ i E_{ab}
ight\} \ \Im \left\{ i E_{ab}
ight\} \end{array} \right]$$

Interpretation:

$$\bar{a}^* = \arg\min_{\bar{a} \in X} ||E_{ab} + iG\bar{a}||^2$$



Solving
$$G\bar{a} = iE_{ab}$$
complex valued

Problem: \bar{a} must be real valued.

$$ar{a} = egin{bmatrix} \Re\left\{G
ight\} \ \cdots \ \Im\left\{G
ight\} \end{bmatrix}^{-1} egin{bmatrix} \Re\left\{iE_{ab}
ight\} \ \cdots \ \Im\left\{iE_{ab}
ight\} \end{bmatrix}$$

Interpretation:

$$\bar{a}^* = \arg\min_{\bar{a} \in X} \|E_{ab} + iG\bar{a}\|^2$$

This is the total energy in the region of interest!

Also related to energy minimization, Borde and Traub (2006)



(Extensions)

$$G\bar{a} = iE_{ab}$$

(Extensions)

$$G\bar{a} = iE_{ab}$$

$$\bar{a} = \begin{bmatrix} \Re \{G\} \\ \cdots \\ \Im \{G\} \end{bmatrix}^{-1} \begin{bmatrix} \Re \{iE_{ab}\} \\ \cdots \\ \Im \{iE_{ab}\} \end{bmatrix}$$

$$\bar{a}^* = \arg\min_{\bar{a} \in X} ||E_{ab} + iG\bar{a}||^2$$

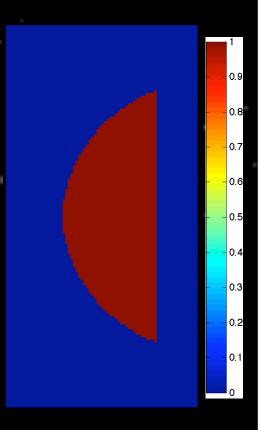


(Extensions)

$$G\bar{a} = iE_{ab}$$

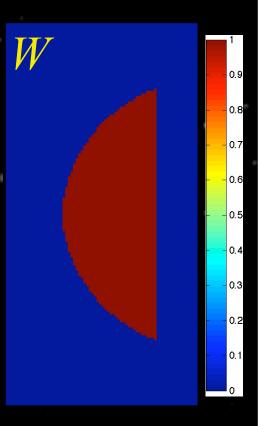
(Extensions)

$$G\bar{a} = iE_{ab}$$

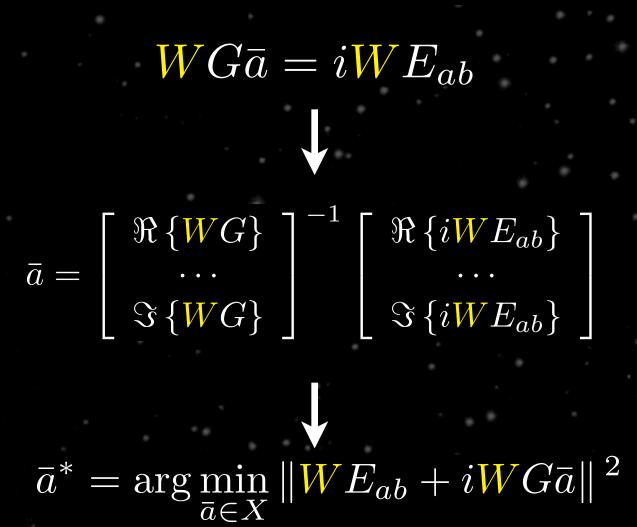


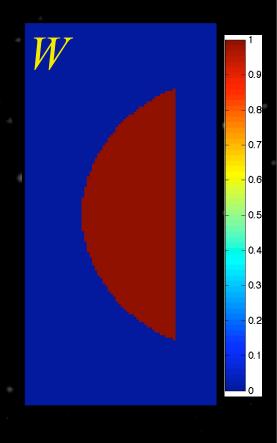
(Extensions)

$$WG\bar{a} = iWE_{ab}$$



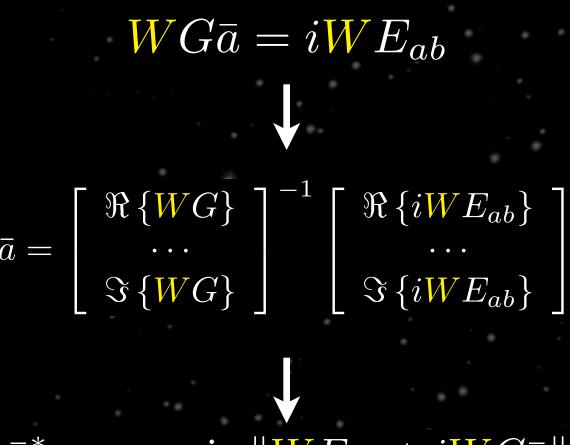
(Extensions)

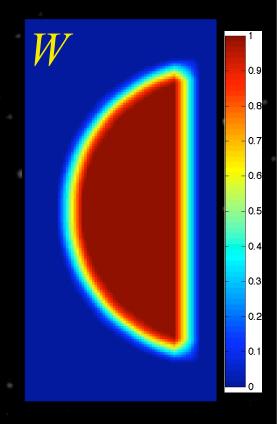




(Extensions)

Extension 1: Image plane pixels weighting

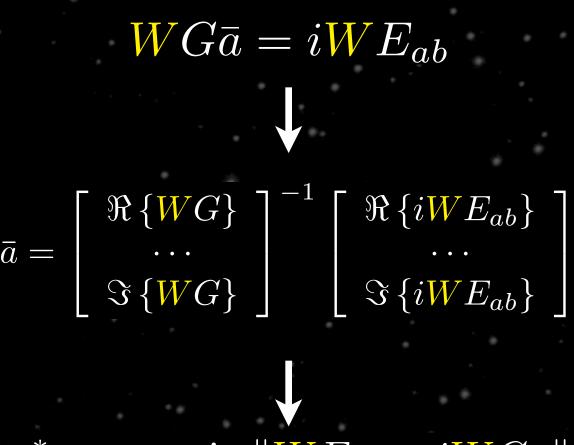


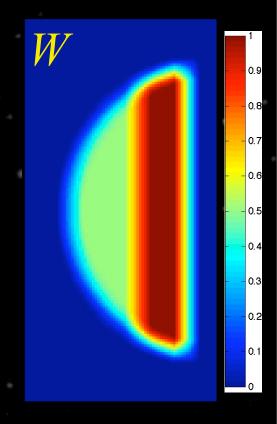


 $\bar{a}^* = \arg\min_{\bar{a} \in X} \| \overline{W} E_{ab} + i \overline{W} G \bar{a} \|^2$

(Extensions)

Extension 1: Image plane pixels weighting





 $\bar{a}^* = \arg\min_{\bar{a} \in X} \|\overline{W}E_{ab} + i\overline{W}G\bar{a}\|^2$



(Extensions)

Extension 1: Image plane pixels weighting

$$WG\bar{a} = iWE_{ab}$$

If the system model is reduced to a single Fourier transform of an infinite aperture and the weighting function W reduces to isolated pixels that change their location according to the brightest peaks in the region of interest, then EFC becomes speckle nulling.

(Extensions)

Extension 2: Actuator regularization (Tikhonov regularization)

$$\bar{a}^* = \arg\min_{\bar{a} \in X} \|G\bar{a} + iE_{ab}\|^2$$

(Extensions)

Extension 2: Actuator regularization (Tikhonov regularization)

$$\bar{a}^* = \arg\min_{\bar{a} \in X} \|G\bar{a} + iE_{ab}\|^2$$

(Extensions)

Extension 2: Actuator regularization (Tikhonov regularization)

$$\bar{a}^* = \arg\min_{\bar{a} \in X} \|G\bar{a} + iE_{ab}\|^2 + \mu^2 \|\bar{a}\|^2$$

(Extensions)

$$\bar{a} = \begin{bmatrix} \Re \left\{ G \\ \Im \left\{ G \right\} \right\} \\ \bar{a} = \begin{bmatrix} \Re \left\{ iE_{ab} \\ \Im \left\{ iE_{ab} \right\} \right\} \\ \end{bmatrix}$$

$$\bar{a}^* = \arg\min_{\bar{a} \in X} \|G(\lambda_1)\bar{a} + iE_{ab}(\lambda_1)\|^2$$

(Extensions)

$$\bar{a} = \begin{bmatrix} \Re \{G(\lambda_1)\} \\ \Im \{G(\lambda_1)\} \end{bmatrix}^{-1} \begin{bmatrix} \Re \{iE_{ab}(\lambda_1)\} \\ \Im \{iE_{ab}(\lambda_1)\} \end{bmatrix}$$

$$\bar{a}^* = \arg\min_{\bar{a} \in X} \|G(\lambda_1)\bar{a} + iE_{ab}(\lambda_1)\|^2$$

(Extensions)

$$egin{array}{c} \Re \left\{ G(\lambda_1)
ight\} \\ \Im \left\{ G(\lambda_1)
ight\} \\ \Re \left\{ G(\lambda_2)
ight\} \\ \Im \left\{ G(\lambda_2)
ight\} \\ \Im \left\{ G(\lambda_2)
ight\} \\ \Im \left\{ iE_{ab}(\lambda_2)
ight\} \\ \Im \left\{ iE_{ab}(\lambda_2)
ight\} \\ rac{1}{\Im \left\{ iE_{ab}(\lambda_2)
ight\}} \\ rac{1}{\Im \left\{ iE_{ab}(\lambda_2)
ight\}} \\ rac{1}{\Im \left\{ iE_{ab}(\lambda_2)
ight\}} \\ \Re \left\{ iE_{ab}(\lambda_k)
ight\} \\ \Im \left\{ iE_{ab}(\lambda_k)
ight\} \\ \Im \left\{ iE_{ab}(\lambda_k)
ight\} \\ rac{1}{\Im \left\{ iE_{ab}(\lambda_k)
ight\}} \\ \frac{1}{\Im \left\{$$

$$\bar{a}^* = \arg\min_{\bar{a} \in X} \|G(\lambda_1)\bar{a} + iE_{ab}(\lambda_1)\|^2$$

(Extensions)

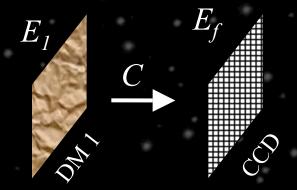
```
\bar{a} = \begin{bmatrix} \Re \left\{ G(\lambda_1) \right\} \\ \Im \left\{ G(\lambda_1) \right\} \\ \Re \left\{ G(\lambda_2) \right\} \\ \Im \left\{ G(\lambda_2) \right\} \\ \Im \left\{ iE_{ab}(\lambda_1) \right\} \\ \Im \left\{ iE_{ab}(\lambda_2) \right\} \\ \Im \left\{ iE_{ab}(\lambda_2) \right\} \\ \vdots \\ \Re \left\{ G(\lambda_k) \right\} \\ \Im \left\{ G(\lambda_k) \right\} \\ \Im \left\{ iE_{ab}(\lambda_k) \right\} \\ \Im \left\{ iE_{ab}(\lambda_k) \right\} \end{bmatrix}
```

$$\bar{a}^* = \arg\min_{\bar{a} \in V} \|G(\lambda_1)\bar{a} + iE_{ab}(\lambda_1)\|^2 + \|G(\lambda_2)\bar{a} + iE_{ab}(\lambda_2)\|^2 + \dots + \|G(\lambda_k)\bar{a} + iE_{ab}(\lambda_k)\|^2$$

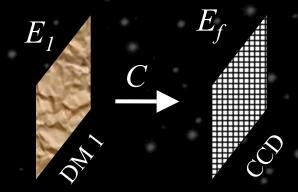


(Extensions)

(Extensions)



(Extensions)



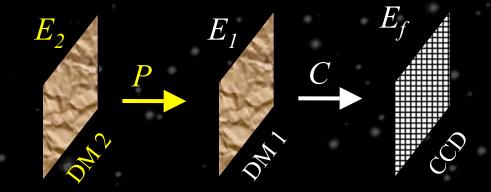
$$\begin{bmatrix} \bar{a_1} \\ \end{bmatrix} = \begin{bmatrix} \Re \{G_1\} \\ \cdots \\ \Im \{G_1\} \end{bmatrix}^{-1} \begin{bmatrix} \Re \{iE_{ab}\} \\ \cdots \\ \Im \{iE_{ab}\} \end{bmatrix}$$

$$\bar{a}^* = \arg\min_{\bar{a} \in X} \|G_1 \bar{a_1}\|$$

$$+E_{ab}||^2$$

(Extensions)

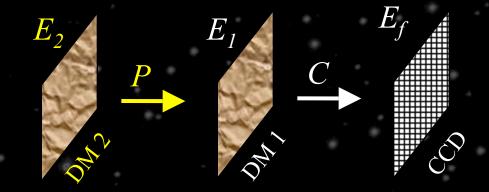
 $+ E_{ab} \|^{2}$



$$\begin{bmatrix} \bar{a_1} \\ \end{bmatrix} = \begin{bmatrix} \Re \{G_1\} \\ \cdots \\ \Im \{G_1\} \end{bmatrix}^{-1} \begin{bmatrix} \Re \{iE_{ab}\} \\ \cdots \\ \Im \{iE_{ab}\} \end{bmatrix}$$

$$\bar{a}^* = \arg\min_{\bar{a} \in X} \|G_1 \bar{a_1}\|$$

(Extensions)



$$\begin{bmatrix} \bar{a_1} \\ \cdots \\ \bar{a_2} \end{bmatrix} = \begin{bmatrix} \Re\{G_1\} & \Re\{G_2\} \\ \cdots & \cdots \\ \Im\{G_1\} & \Im\{G_2\} \end{bmatrix}^{-1} \begin{bmatrix} \Re\{iE_{ab}\} \\ \cdots \\ \Im\{iE_{ab}\} \end{bmatrix}$$

$$\bar{a}^* = \arg\min_{\bar{a} \in X} \|G_1 \bar{a_1} + G_2 \bar{a_2} + E_{ab}\|^2$$



(All extensions)

Basic EFC

```
\Re \left\{ 
ight.
                                                                                                                                                                                                       E_{ab}
```



(All extensions)

Image plane pixels weighting

```
\Re \{WG
                                                                                           \Re \left\{ iWE_{ab} \right\}
\Im \{WG
                                                                                           \Im \{iWE_{ab}\}
```



(All extensions)

Actuators regularization

```
\Re \{WG
      \Im \{WG
\mu_1
```

```
\Re \left\{ iWE_{ab} \right\}
\Im \{iWE_{ab}\}
```



(All extensions)

Multi wavelength correction

```
\Re \left\{ \overline{WG} \ \overline{(\lambda_1)} \right\}
                 \Im \left\{ WG \ \overline{(\lambda_1)} \right\}
                 \Re \left\{ WG \ \overline{(\lambda_2)} \right\}
                 \Im\left\{\overline{WG}\left(\lambda_{2}
ight)
ight\}
                 \Re\left\{\overline{WG}\left(\lambda_k\right)\right\}
                 \Im \left\{ \overline{WG} \left( \lambda_k \right) \right\}
\mu_1
```

```
\Re\left\{iWE_{ab}(\lambda_1)\right\}
\Im \left\{ iW\overline{E_{ab}(\lambda_1)} \right\}
\Re\left\{iW\overline{E_{ab}(\lambda_2)}\right\}
\Im \left\{ iWE_{ab}(\lambda_2) \right\}
\Re \left\{ iWE_{ab}(\lambda_k) \right\}
\Im\left\{iW\overline{E_{ab}(\lambda_k)}\right\}
```



(All extensions)

Multi DM correction

```
\Re \left\{ \overline{WG_1(\lambda_1)} \right\}
                                                                                                                  \Re\left\{WG_2(\lambda_1)\right\}
                                           \Im \left\{ WG_1(\lambda_1) \right\}
                                                                                                                  \Im \left\{ WG_2(\lambda_1) \right\}
                                           \Re\left\{\overline{W}G_1(\lambda_2)\right\}
                                                                                                                  \Re \left\{ \overline{WG_2(\lambda_2)} \right\}
                                           \Im\left\{WG_1(\lambda_2)\right\}
                                                                                                                  \Im\left\{WG_2(\lambda_2)\right\}
\bar{a}_1
                                                                                                                  \Re\left\{W\overline{G_2(\lambda_k)}\right\}
                                           \Re\left\{WG_1(\lambda_k)\right\}
\bar{a}_2
                                           \Im \left\{ WG_1(\lambda_k) \right\}
                                                                                                                  \Im\left\{WG_2(\lambda_k)\right\}
                              \mu_1
                                                                                                     \mu_2
```

```
\Re\left\{iWE_{ab}(\lambda_1)\right\}
\Im \left\{ iW\overline{E_{ab}(\lambda_1)} \right\}
\Re \left\{ iWE_{ab}(\lambda_2) \right\}
\Im \left\{ iWE_{ab}(\lambda_2) \right\}
\Re \left\{ iWE_{ab}(\lambda_k) \right\}
\Im \left\{ iWE_{ab}(\lambda_k) \right\}
```



(The reconstruction stage)

The electric field at the science camera is approximated as:

$$E_f \approx E_{ab} + iC \{A\psi\}$$



(The reconstruction stage)

The electric field at the science camera is approximated as:

$$E_{\mathbf{k}} \approx E_{ab} + iC \left\{ A \psi_{\mathbf{k}} \right\}$$



(The reconstruction stage)

The electric field at the science camera is approximated as:

$$E_{\mathbf{k}} \approx E_{ab} + iC \left\{ A\psi_{\mathbf{k}} \right\}$$

$$\downarrow$$

$$I_{\mathbf{k}} \approx |E_{ab} + iC \left\{ A\psi_{\mathbf{k}} \right\}|^{2}$$



(The reconstruction stage)

The electric field at the science camera is approximated as:

$$E_{\mathbf{k}} \approx E_{ab} + iC \left\{ A\psi_{\mathbf{k}} \right\}$$

$$\downarrow$$

$$I_{\mathbf{k}} \approx |E_{ab} + iC \left\{ A\psi_{\mathbf{k}} \right\}|^{2}$$

Suppose we deform the DM in pairs of shapes, ψ_k and $-\psi_k$, then,



(The reconstruction stage)

The electric field at the science camera is approximated as:

$$E_{\mathbf{k}} \approx E_{ab} + iC \left\{ A\psi_{\mathbf{k}} \right\}$$

$$\downarrow$$

$$I_{\mathbf{k}} \approx |E_{ab} + iC \left\{ A\psi_{\mathbf{k}} \right\}|^{2}$$

Suppose we deform the DM in pairs of shapes, ψ_k and $-\psi_k$, then,

$$I_k^+ \approx |E_{ab} + iC \{A\psi_k\}|^2$$
$$I_k^- \approx |E_{ab} - iC \{A\psi_k\}|^2$$



(The reconstruction stage)

If we take the difference between those images...



(The reconstruction stage)

If we take the difference between those images...

$$I_k^+ - I_k^- = 2\overline{E_{ab}}C\left\{A\psi_k\right\} + 2E_{ab}\overline{C\left\{A\psi_k\right\}}$$

(The reconstruction stage)

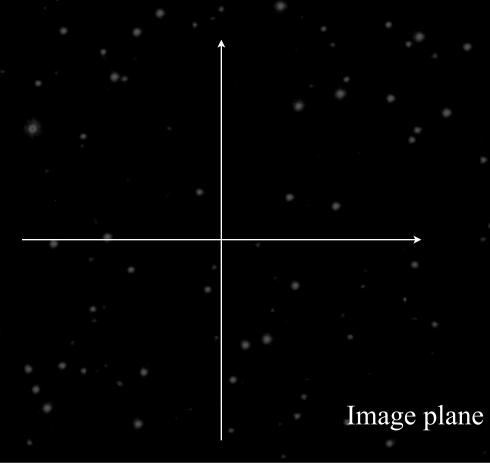
If we take the difference between those images...

$$I_k^+ - I_k^- = 2\overline{E_{ab}}C\left\{A\psi_k\right\} + 2E_{ab}\overline{C\left\{A\psi_k\right\}}$$

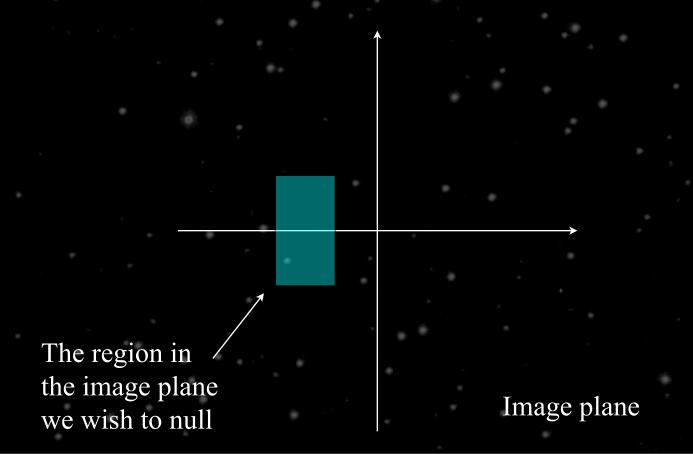
If we take more pairs and stack them up...

$$\begin{bmatrix} I_1^+ - I_1^- \\ \vdots \\ I_k^+ - I_k^- \end{bmatrix} = \frac{1}{4} \begin{bmatrix} \Re\{C\{A\psi_1\}\} & \Im\{C\{A\psi_1\}\} \\ \vdots & \vdots \\ \Re\{C\{A\psi_k\}\} & \Im\{C\{A\psi_k\}\} \end{bmatrix} \begin{bmatrix} \Re\{E_{ab}\} \\ \Im\{E_{ab}\} \end{bmatrix}$$









$$DM(x,y) = ?$$

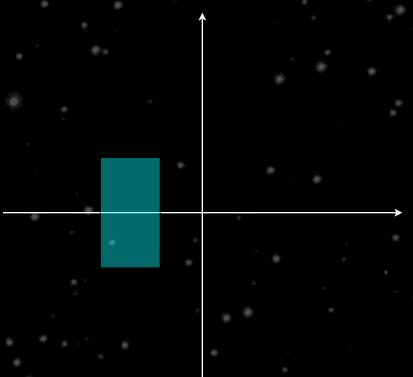


Image plane

$$DM(x,y) = sinc(w_x x) sinc(w_y y)$$

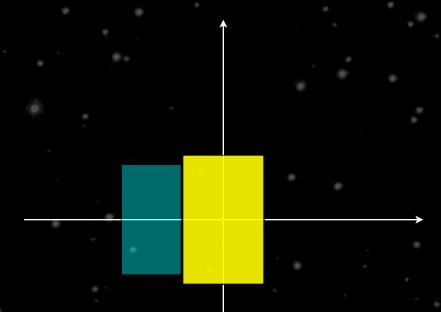
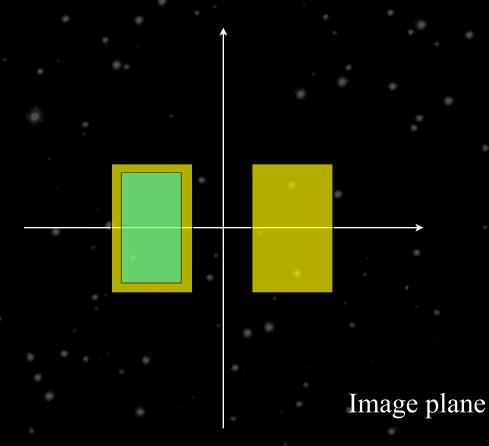
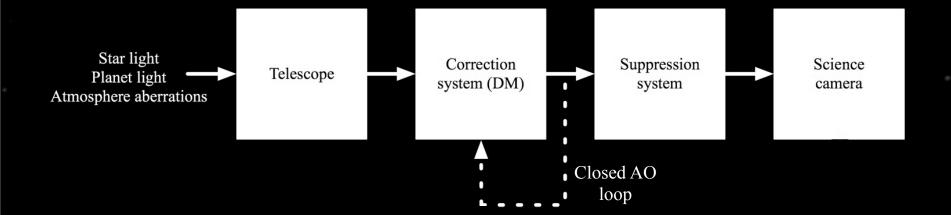


Image plane

$$DM(x,y) = sinc(w_x x) sinc(w_y y) cos(f_x x + \phi)$$

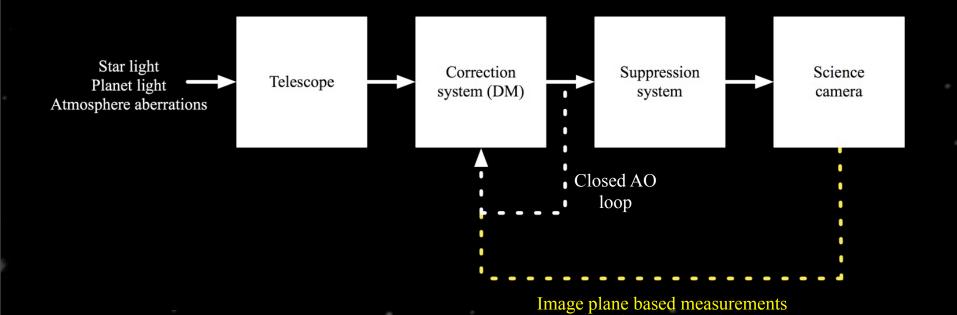


Can we all live in peace?

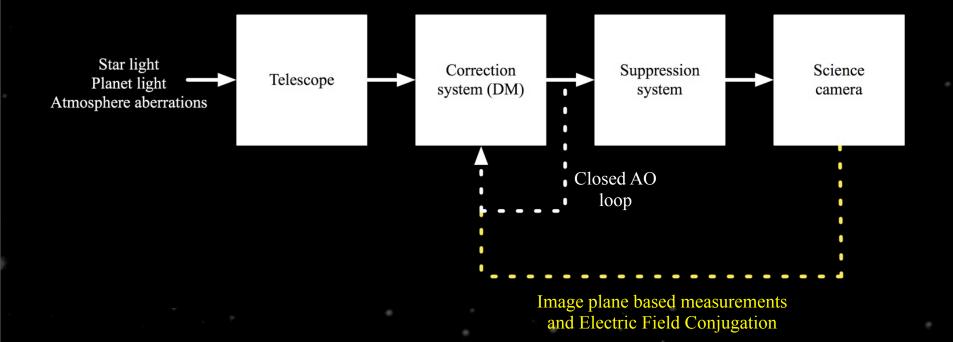


Can we all live in peace?

and Electric Field Conjugation



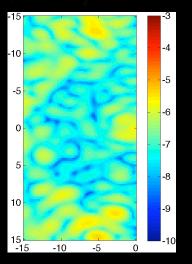
Can we all live in peace?



Experiments have started at Palomar to try and close the image plane based correction loop around the AO system.

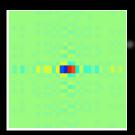


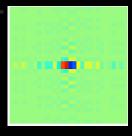
Measured intensity

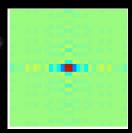


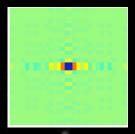


DM voltages

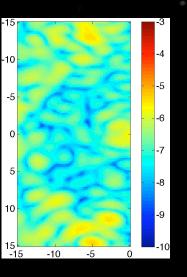




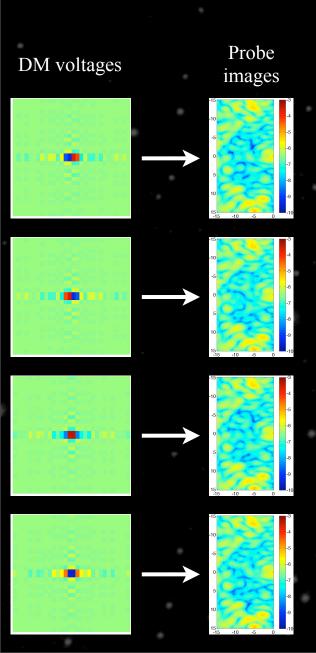




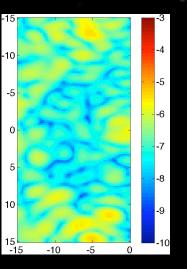
Measured intensity

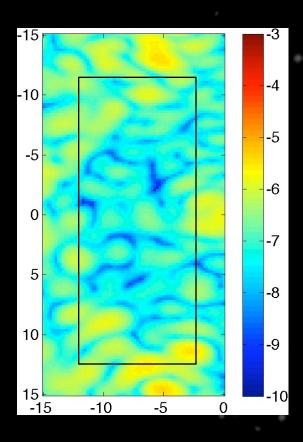


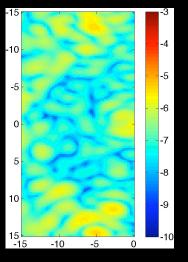




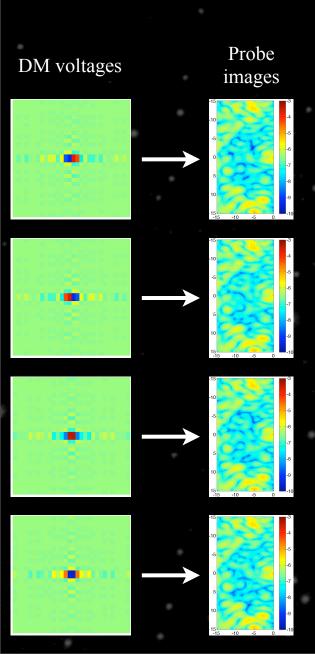
Measured intensity

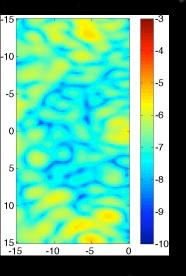




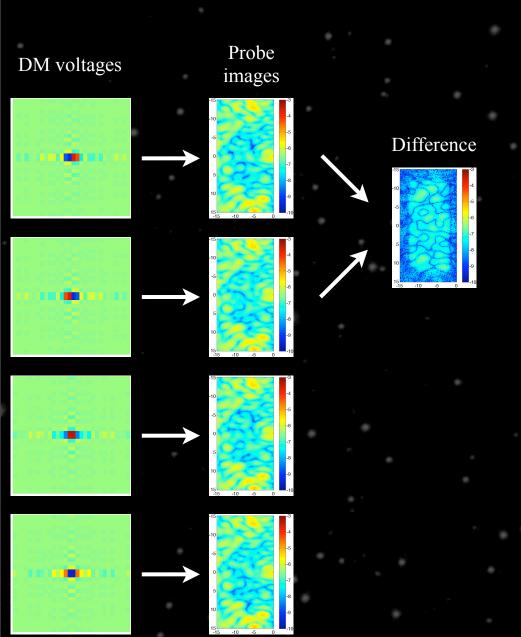


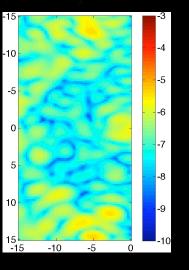




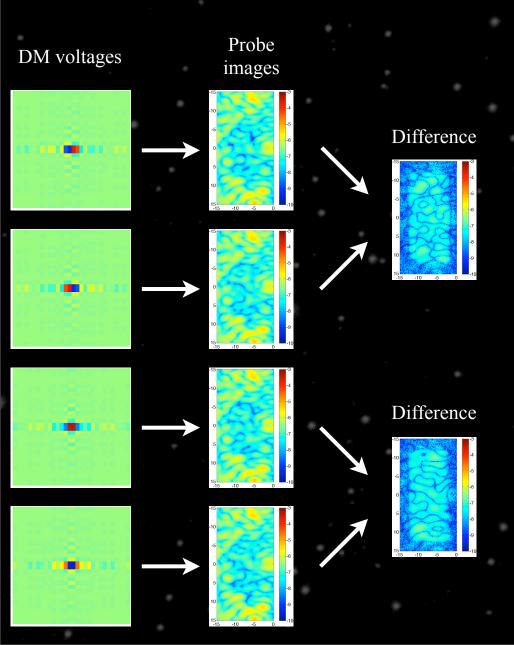


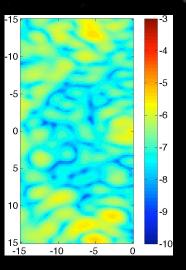




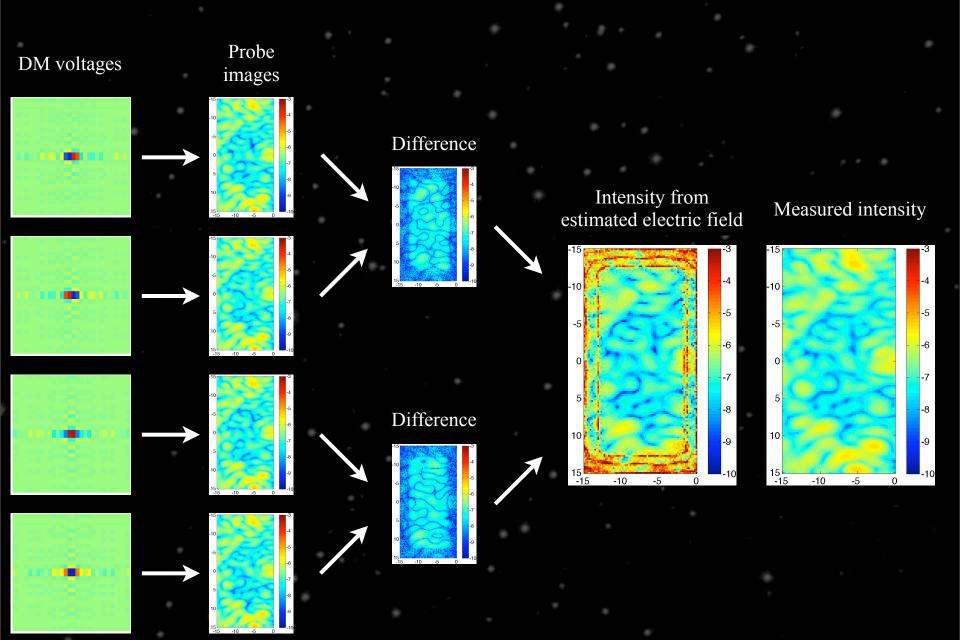




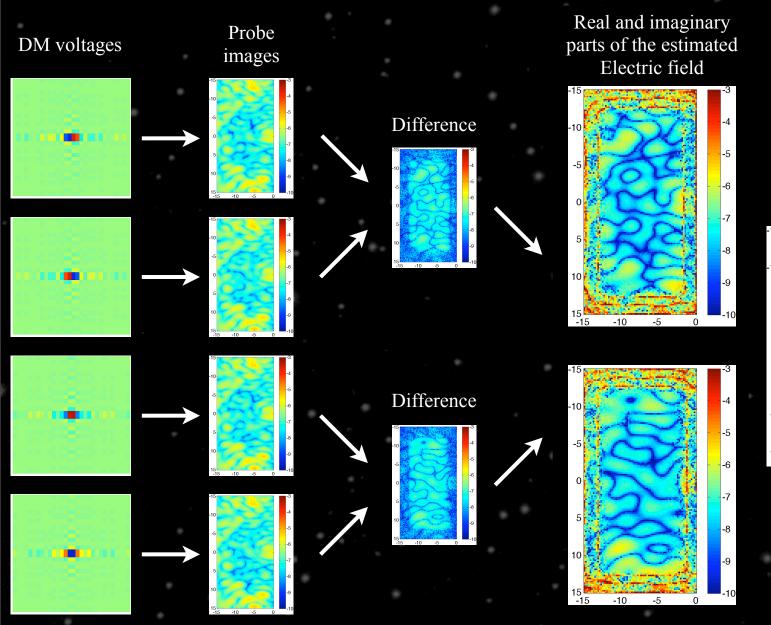


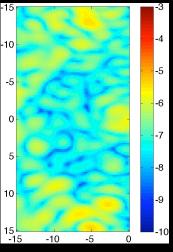




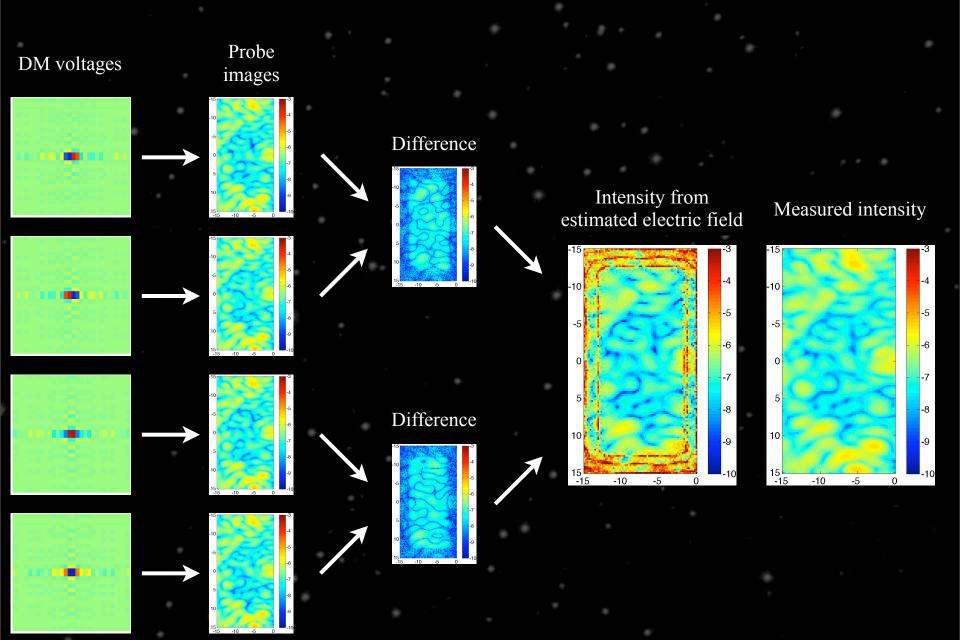




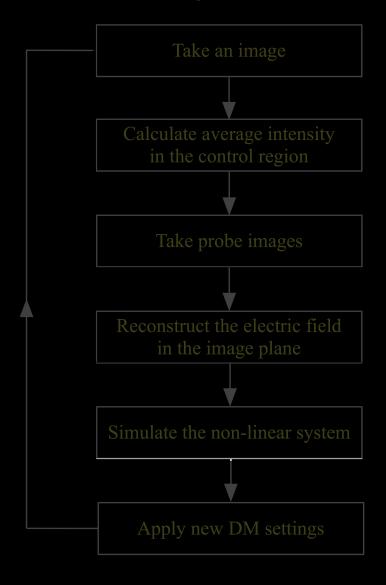




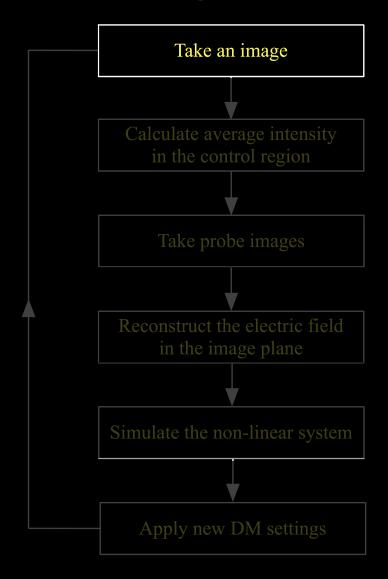




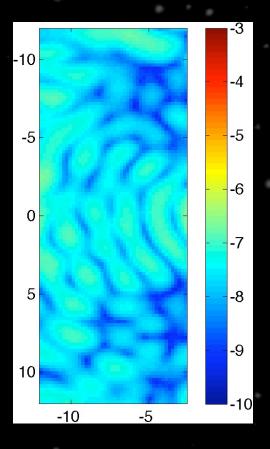




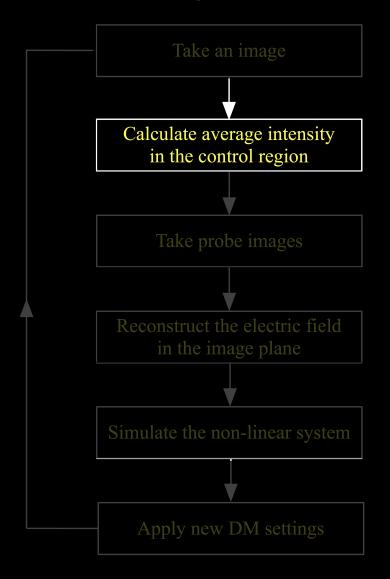




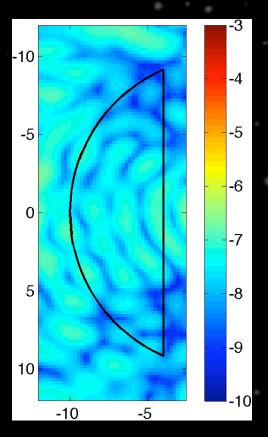




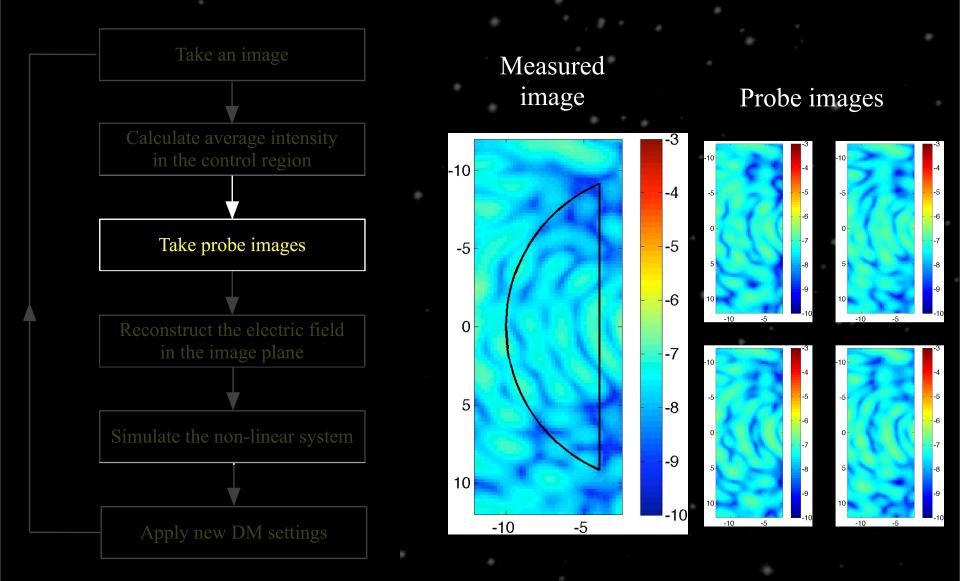




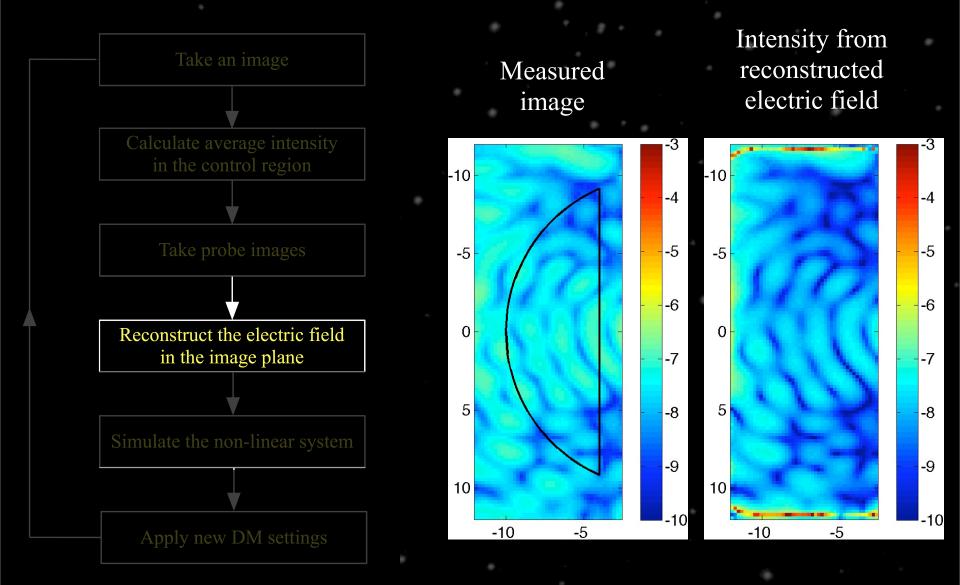




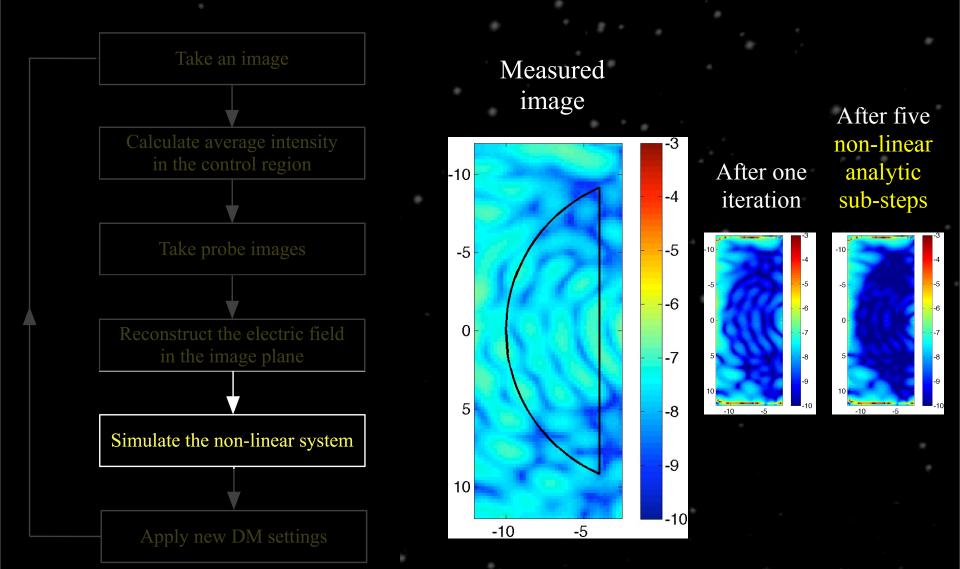




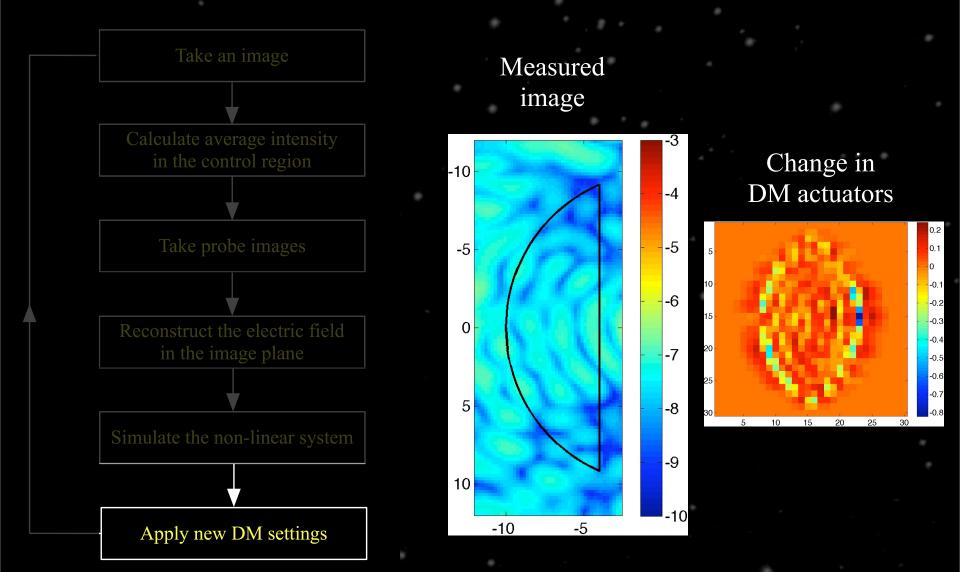




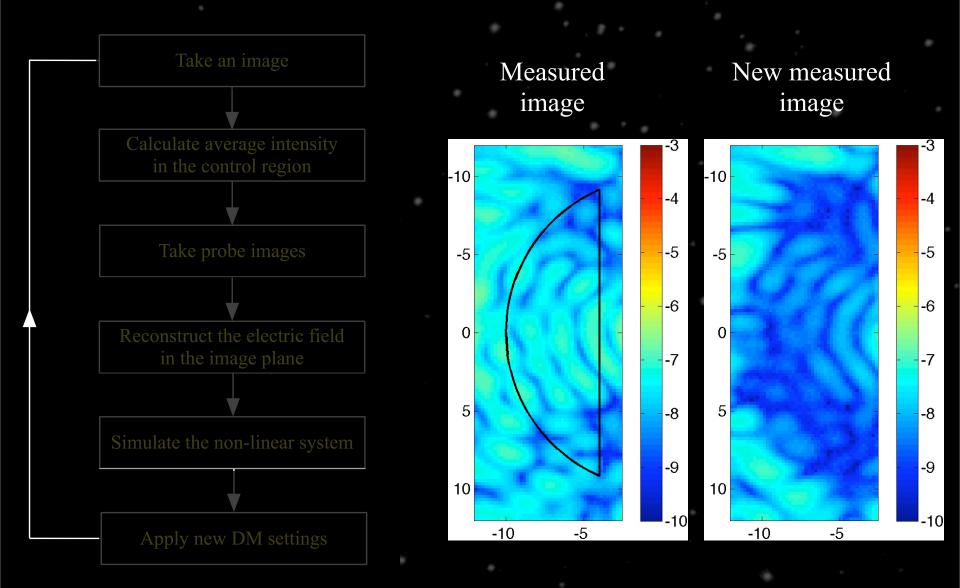








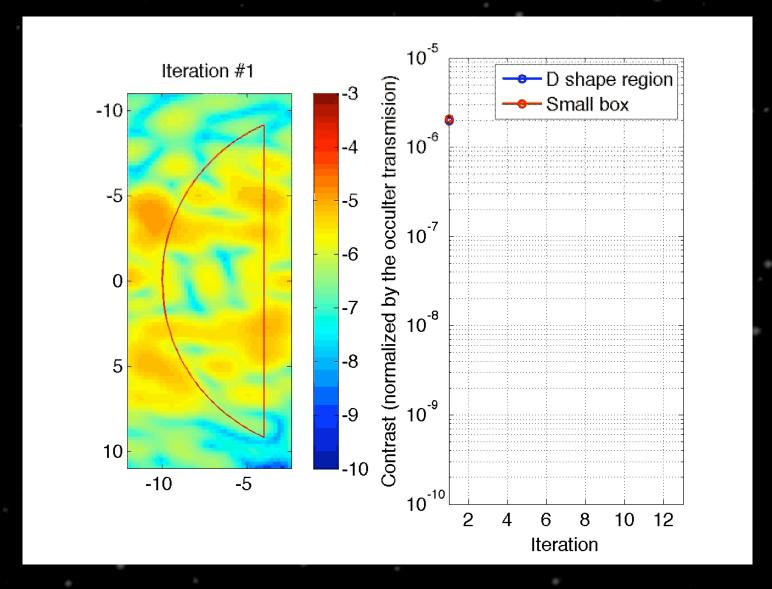






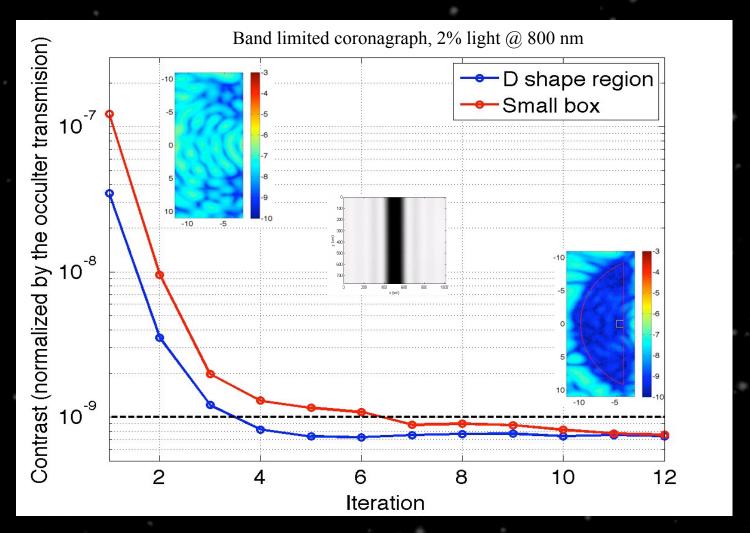
Band limited coronagraph, 2% light around 800nm

Band limited coronagraph, 2% light around 800nm



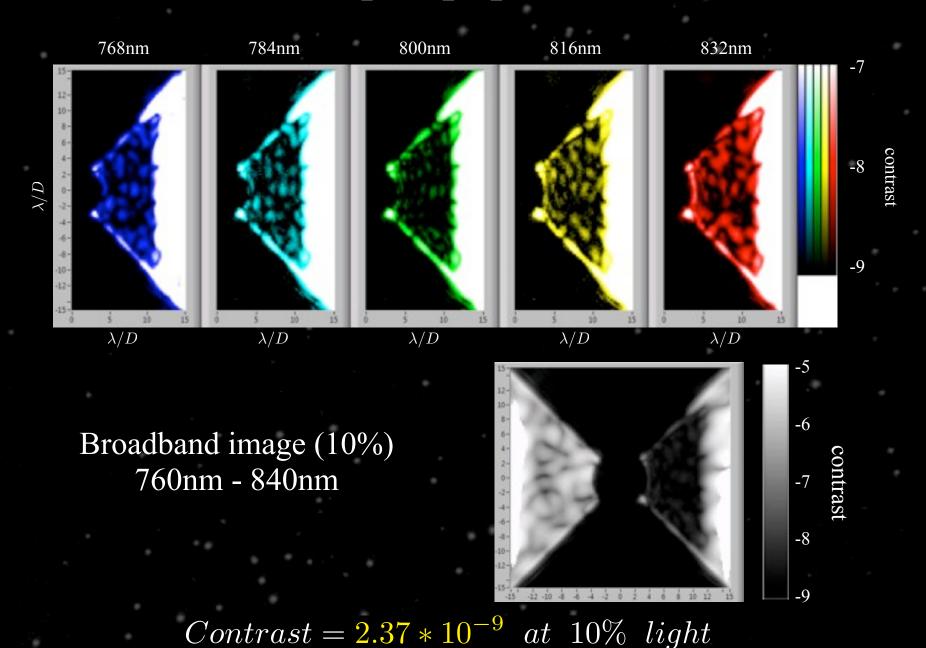


Band limited coronagraph, 2% light around 800nm



 $Contrast = 6 * 10^{-10}$ at 2% light $Contrast = 4 * 10^{-10}$ at monochromatic light

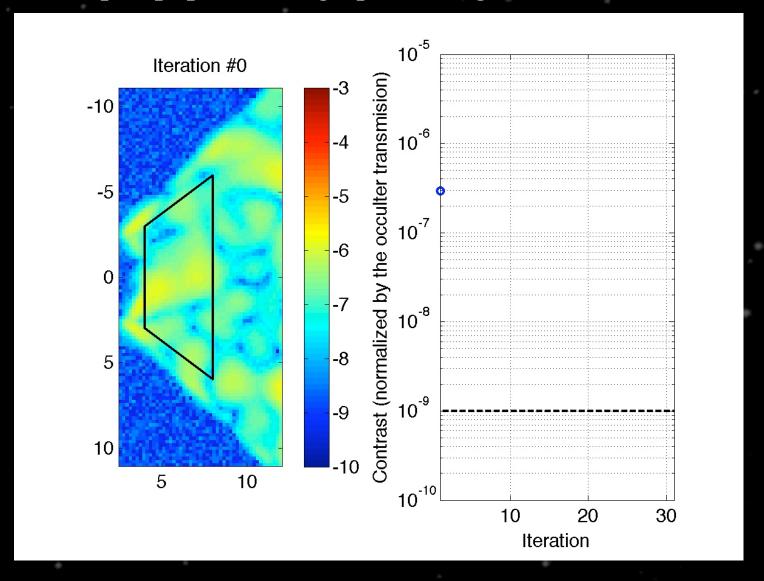
Shaped pupils are achromatic...





Shaped pupil coronagraph, 2% light around 800nm

Shaped pupil coronagraph, 2% light around 800nm



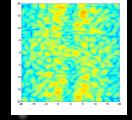


(Multi wavelength correction results)



(Multi wavelength correction results)

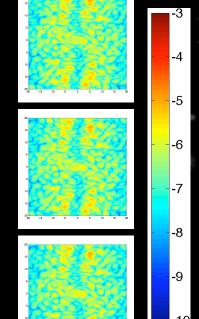




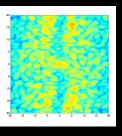


800nm

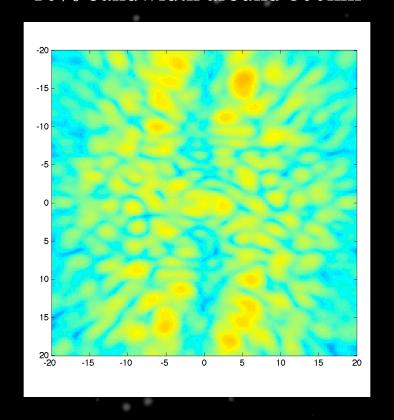
816nm



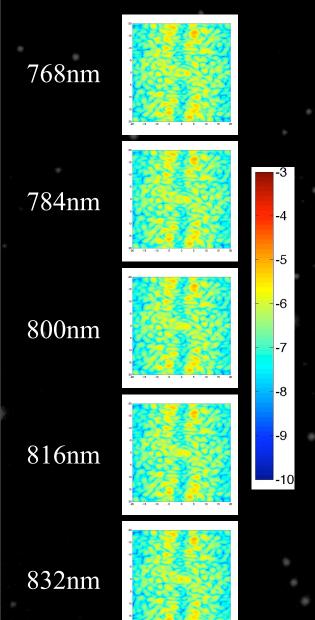
832nm

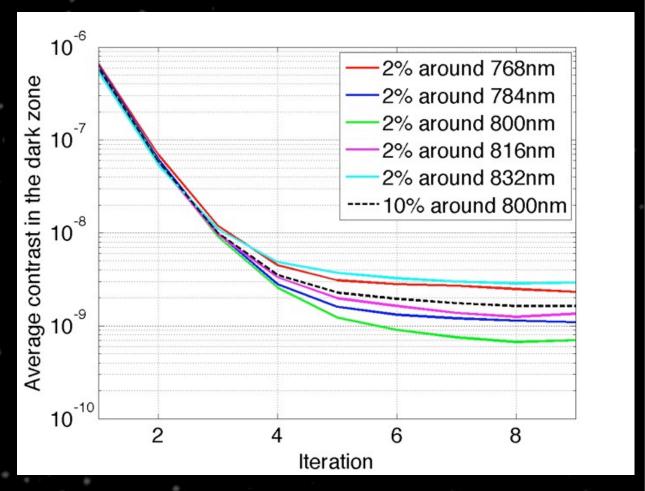


10% bandwidth around 800nm



(Multi wavelength correction results)





 $Contrast = 2.3 * 10^{-9} at 10\% light$



Electric Field ConjugationSummary

- The reconstruction method using pairs of images has been implemented at HCIT and has been an important part of the success of reaching new record contrasts in both narrow and broad bands.
- The EFC correction algorithm has been successfully implemented at HCIT, working with both band limited coronagraphs and shaped pupils coronagraphs.
- Record contrasts have been achieved at HCIT and the Princeton testbed using these methods for reconstruction and correction.
- Future work includes implementation of the algorithm at Palomar to run along side the AO system to correct for quasi-static and static speckles.